2016 IEEE 19th International Conference on Intelligent Transportation Systems (ITSC)
Windsor Oceanico Hotel, Rio de Janeiro, Brazil, November 1-4, 2016

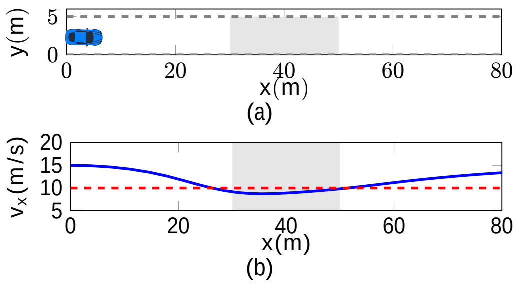

Fig. 1: Speed bump scenario: (a) illustration of the scenario, (b) longitudinal speed pro le w.r.t. the longitudinal offset.

desired binary state, and therefore the term jj (k) $_r$(k)jj^2_S in the generic formulation is ignored.

In all cases, the ego vehicle is assumed to start at position (0; 2:5) in the rightmost lane. The initial and desired speeds are equal, with $v_x(0)$ = v_r = 15 m=s. The width of each lane is 5 m. Other parameters that are common in all cases are recapitulated in Table I. Scenario-speci c parameters will be presented respectively in each case study.

We use the commercial solver Gurobi [19] to compute solutions to our MIQP formulation. Simulation codes are written in Python, and experiments are performed on a laptop with Intel Core i5-5300U CPU clocked at 2.30GHz with 8GB RAM.

A. Speed bump

The rst case study considers the speed bump scenario (Fig. 1a) that is used as an example in II-B.

The speed bump conditions are given in (6). In the simulation, we use a prediction horizon T = 5 s and = 0:25 s Fig. 1b illustrates the longitudinal speed pro le of the planned trajectory with respect to the traveled distance. We observe that vehicles effectively reduces its speed to less than 10 m=s within the interval of [30; 50].

B. Obstacle avoidance

We now consider an obstacle avoidance scenario during on-road driving. The irregular shapes of obstacles are approximated using minimal bounding rectangles. A more complex polygonal modeling is also possible, at the cost of increased computational complexity. For an obstacle with bounding rectangle [x (k) L ; x (k) + L] [y (k) W ; y (k) + W], the set of constraints for collision avoidance is then given as 8k 0,

$$_1(k) = 1 \ ) \ x(k) \quad x\ (k) \quad L\ ; \quad (10a)$$

$$_2(k) = 1 \ ) \ x(k) \quad x\ (k) + L\ ; \quad (10b)$$

$$_3(k) = 1 \ ) \ x(k) \quad y\ (k) \quad W\ ; \quad (10c)$$

$$_4(k) = 1 \ ) \ x(k) \quad y\ (k) + W\ ; \quad (10d)$$

$$_1(k) + \ _2(k) + \ _3(k) + \ _4(k) = 1 : \quad (10e)$$

Note that the formulation allows both moving and still obstacles. The conditions (10) state that the vehicle must be separated from the obstacle, either by a longitudinal distance L or laterally by W .

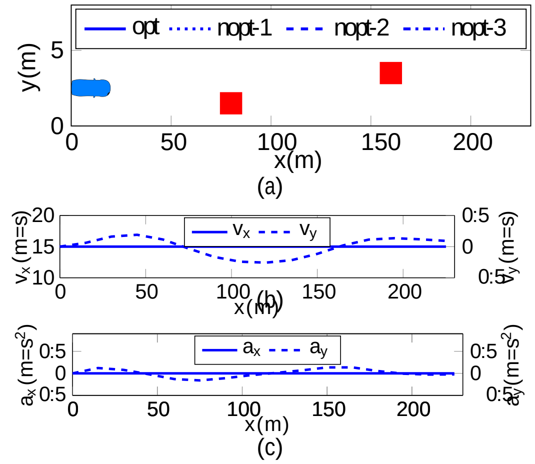

Fig. 2: Obstacle avoidance scenario: (a) the globally optimal trajectory and locally optimal trajectories (respective costs: opt 3.16, nopt-1 36.5, nopt-2 37.2, nopt-3 84.67), (b) speed pro les, (c) acceleration pro les. Obstacles are marked in red.

For illustration, we consider two identical obstacles centered at (80; 1:5) and (160; 3:5). Parameters are T = 15 s, = 1 s, L = 10 m and W = 2 m. The trajectory labeled opt in Fig. 2a is the global optimum found by the MIQP planner; for comparison purposes, we also plot all locally optimal trajectories nopt-1,2,3 with their respective costs. We observe that our method can effectively nd the globally optimal trajectory. Fig. 2b and 2c respectively illustrate the longitudinal and lateral speed and acceleration pro les of the ego vehicle.

C. Overtaking in a two-lane road

This case study considers an overtaking scenario on a two-lane road with oncoming traf c. Fig. 3a shows the initial con gurations of nearby vehicles, all driving with a constant speed of 10 m=s. This scenario is considered as dif cult for both human drivers and autonomous vehicles [8], [17]. Reference [10] shows the existence of multiple homotopy classes in this scenario and proposes to exhaustively search for the globally optimal solution. Here, we show that we can nd the globally optimal solution without explicitly enumerating all homotopy classes.

It is possible to model surrounding vehicles as rectangles as in the previous case study, thus requiring four integer variables for each vehicle and each time step k. However, by introducing the so called ramp barrier [8], [17], the problem can be further simpli ed by approximating the rectangular obstacle region by a triangle only using two linear constraints as shown in Fig. 3a.

Let be a surrounding vehicle; if is in the same lane as the ego vehicle, the constraints are given as

$$(k) = 0 \ ) \ \frac{x(k) \quad x\ (k)}{L} + \frac{y(k) \quad y\ (k)}{W} \quad 1; \quad (11a)$$

$$(k) = 1 \ ) \ \frac{x(k) \quad x\ (k)}{L} + \frac{y(k) \quad y\ (k)}{W} \quad 1; \quad (11b)$$

where L and W are minimal longitudinal and lateral separations during lane change. Similarly, the constraints for

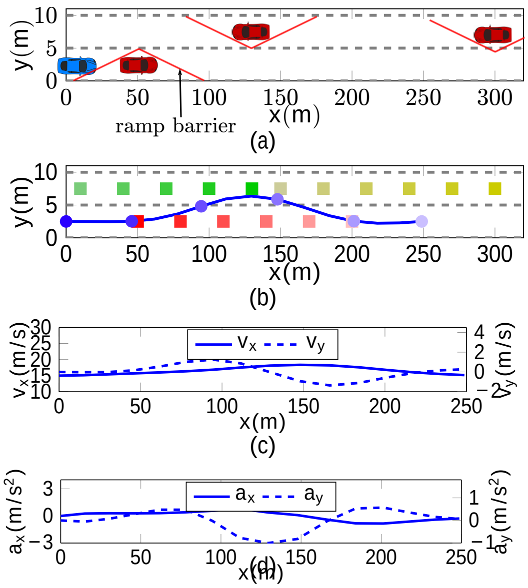

Fig. 3: Overtaking scenario: (a) illustration of the scenario and the ramp barrier methods, (b) the overtaking trajectory, (c) speed pro les, (d) acceleration pro les. The position of the ego vehicle is shown as blue dots, and rectangles with different colors mark the positions of surrounding vehicles. Time dimension is color-coded, with lighter colors corresponding to instants further away in time.

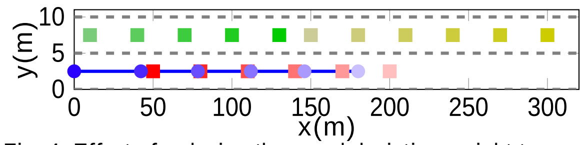

Fig. 4: Effect of reducing the speed deviation weight to $q_1 = 0.5$ in the overtaking scenario. The position of the ego vehicle is shown as blue dots, and rectangles with different colors mark the positions of surrounding vehicles. Time dimension is color-coded, with lighter colors corresponding to instants further away in time.

an oncoming vehicle can be modeled as

$$(k) = 0 \;) \quad \frac{x(k) \quad x\ (k)}{L} + \frac{y(k) \quad y\ (k)}{W} \quad 1; \quad (12a)$$

$$(k) = 1 \;) \quad \frac{x(k) \quad x\ (k)}{L} + \frac{y(k) \quad y\ (k)}{W} \quad 1: \quad (12b)$$

Using this formulation, only one integer variable is required per vehicle and per time step.

In the simulation, we adopt a prediction horizon of T = 15 s and we let = 1 s. To allow the ego vehicle to temporarily cross the lane border, the upper limit $\bar{y}$ is relaxed. Fig. 3b illustrates the trajectory of overtaking. We observe that the ego vehicle decides to accelerate slightly so that it can use the space between the rst oncoming vehicle and the second oncoming vehicle to perform the overtaking.

For comparison purposes, we reduce the penalty on speed deviation q_1 to 0:5 in the cost function (7). The resulting trajectory is shown in Fig. 4: in this case, the ego vehicle chooses not to overtake, as the cost of this maneuver is higher than that of following the slower car, due to the small penalty on the speed deviation. This demonstrates the exibility of the MIQP formulation: different driving styles can be con gured simply by modulating the weighting terms.

D. Lane change

This last case study considers the problem of decision making and trajectory generation for a lane change maneuver: the ego vehicle must decide the objective lane as well as the optimal trajectory to reach this lane, without colliding with surrounding vehicles. The complexity of this problem lies in the multiple discrete choices raised from multiple lanes and multiple vehicles on each lane. References [15], [16] have considered this problem using MILP formulations; however, their modeling cannot ensure that trajectories are dynamically feasible due to important simpli cations of the vehicle dynamics.

We consider a road with N lanes, labeled by 2 f 1; :::; N g. We introduce a binary variable (k) that equals 1 if the ego vehicle is on lane at time step k. Let V be the set of surrounding vehicles and V be the set of vehicles inside lane . We introduce the following logical constraint: 8k > 0;

$$(k) = 1 \;) \quad y_r(k) = y \; \wedge \; y(k) \; 2 \; [\underline{y} \; ; \bar{y} \;] \; ; \quad (13)$$

such that, if the ego vehicle is in lane , then the vehicle should be within the boundary of lane and the reference centerline should be set to the centerline of the lane.

Moreover, we add the following collision avoidance constraints: 8k 0;

$$(k) = 1 \;) \; 8 \quad 2 \; V \; ; \quad (k) = 1 \;) \; x(k) \quad x\ (k) \quad L \; ;$$
$$(k) = 0 \;) \; x(k) \quad x\ (k) + L \; ; \quad (14)$$

such that the ego vehicle must avoid collisions with all the vehicles in lane .

The ego vehicle is only allowed to be in one lane at any given time, thus we add the following constraint: 8k 0,

$$\sum_{=1}^{N} (k) = 1 : \quad (15)$$

Fig. 5a shows a highway with three lanes. The ego vehicle starts in the rightmost lane with a speed of 15 m=s. Surrounding vehicles are distributed over three lanes. The vehicle on the leftmost lane drives at a speed of 15 m=s while other surrounding vehicles drive at a speed of 10 m=s. Constraints (13), (14) and (15) are enforced along with other constraints on the formulated MIQP problem. The horizon is set to T = 15 s and the time step = 1 s. The constraint on lane boundary is temporarily deactivated. We observe in Fig. 5b that the ego vehicle chooses the left-most lane as the objective lane and plans a dynamically feasible and collision-free trajectory to reach the lane within the prediction horizon.

Table II presents the computation times for the four case studies, demonstrating the real-time capability of the proposed formulation; future work will focus on further reducing this computation time.

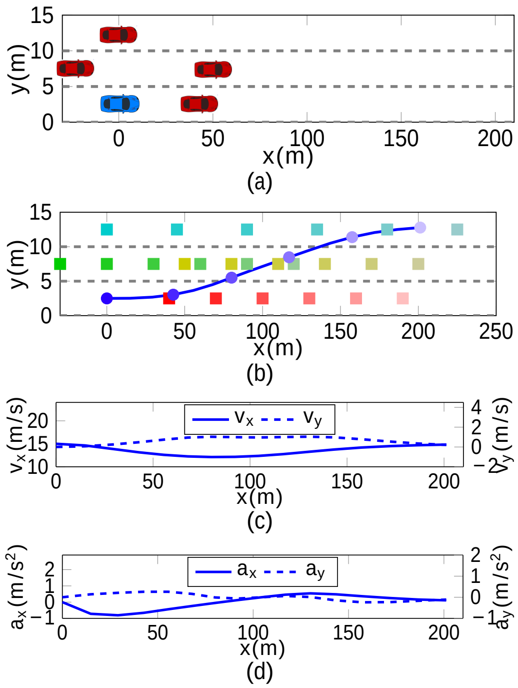

Fig. 5: Lane change scenario: (a) illustration of the scenario, (b) trajectory of the ego vehicle and the surrounding vehicles, (c) speed pro les, (d) acceleration pro les. The position of the ego vehicle is shown as blue dots, and rectangles with different colors mark the positions of surrounding vehicles. Time dimension is color-coded, with lighter colors corresponding to instants further away in time.

Speed bump	Obstacle avoidance	Overtaking	Lane change
17 ms	73 ms	81 ms	228 ms

TABLE II: Time statistics

IV. D