Optimal Trajectory Planning for Autonomous Driving Integrating Logical Constraints: A MIQP Perspective

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Introduction

Formulation

Example Applications

Conclusions
Sales, travel and fleet projections of autonomous vehicles

* T. Lidman, Autonomous Vehicle Implementation Predictions, presented in TRB 2015
Motion planning

Generation of feasible (or preferably optimal) trajectories for autonomous vehicles

*w.r.t.* certain performance index

subj. to constraints raised from operation limits and the environment.

*A ride in Google car, acquired from youtube.*
Motion Planning methods

- **Sampling based methods**
  - Deterministic sampling: A*, State lattices
  - Stochastic sampling: RRT, RRT*

- **Optimization based methods**: Model Predictive Control

*Figures are cited from various publications: Ziegler et al, Werling et al, Xu et al, Frazzoli et al, Ziegler et al.*
Optimization based methods

Generation optima trajectories

w.r.t. certain performance index

subj. to constraints raised from
operation limits and the
environment.

Compute $x^*$

$$\min_{u} J(x, u),$$

$x(k + 1) = f^d(x(k), u(k)), \forall k \in [0, ..., K - 1]$,  
$x(k) \in \mathcal{X}, u(k) \in \mathcal{U}, \forall k \in [0, ..., K - 1]$,  
$x(0) = x^0$.  

Introduction
Optimization based methods

- Solvers: continuous and local methods like Interior-Point method, Sequential Quadratic Programming

- Advantage
  - use gradient information to quickly converge to local optimum
  - able to systematically handle various (continuous and differentiable) constraints
Logical constraints in on-road driving

- If the vehicle is on a speed bump, then its speed must be reduced to 40 km/h
- If the vehicle is approaching the exit of highway, then it must be on the exit lane
- If a vehicle is overtaking, then the ego vehicle must decelerate to facilitate it.
- Multiple maneuver variants

*Figure from Bender et al*
Can current motion planning algorithms handle logical constraints?

- Sampling based approaches are by nature compatible with logical constraints, however, sub-optimal

- Optimization based approaches
  - cannot handle dis-continuous or non-differentiable constraints
  - can be trapped in local optimum
  - Heuristic exists to approximate dis-continuous or non-differentiable constraints by nonlinear differentiable constraints or to initialize the solver near the presumably global optimum.
  - However...

Motivation for the development of a globally optimal algorithm capable of handling logical constraints
Linear point-mass model

- Assumption: the reference path is considered as straight in the prediction horizon of the MPC so that we can set the $x$-axis of the Cartesian frame as the longitudinal direction of the reference path

\[
x(t) = [x(t), v_x(t), a_x(t), y(t), v_y(t), a_y(t)]^T,
\]

- Define

\[
u(t) = [j_x(t), j_y(t)]^T
\]

- State transition equation

\[
\begin{align*}
\dot{x} &= v_x, & \dot{y} &= v_y, \\
\dot{v}_x &= a_x, & \dot{v}_y &= a_y, \\
\dot{a}_x &= j_x, & \dot{a}_y &= j_y.
\end{align*}
\]
Linear point-mass model

- Limit on heading

\[ v_y \in [v_x \tan(\theta), v_x \tan(\bar{\theta})]. \]

- Limit on yaw

\[ a_y \in [-v_x \bar{\omega}, v_x \omega]. \]
Logic for driving

• Example: if the vehicle’s longitudinal position is between 30m and 50m, then its speed must be less than 10 m/s

• Formal description using propositional logic:

\[ P_1 = [x(k) \geq 30], \]
\[ P_2 = [x(k) \leq 50], \]
\[ P_3 = [v_x(k) \leq 10] \]

\[ \forall k \geq 0, (P_1 \land P_2) \Rightarrow P_3 \]
Logic for driving

• Associate binary variable to propositional logic predicates

\[ P_1 = [x(k) \geq 30], \]
\[ P_2 = [x(k) \leq 50], \]
\[ P_3 = [v_x(k) \leq 10] \]

\[ \forall k \geq 0, (P_1 \land P_2) \Rightarrow P_3 \]

\[ \delta_1(k) = 1 \iff x(k) \geq 30, \]
\[ \delta_2(k) = 1 \iff x(k) \leq 50, \]
\[ \delta_3(k) = 1 \iff v_x(k) \leq 10, \]
\[ -\delta_1(k) + \delta_3(k) \leq 0, \]
\[ -\delta_2(k) + \delta_3(k) \leq 0, \]
\[ \delta_1(k) + \delta_2(k) - \delta_3(k) \leq 1. \]

• Big-M method:

\[ \delta_1(k) = 1 \iff x(k) \geq 30, \]

\[ x(k) \geq 30 - M(1 - \delta_1(k)), \]
\[ x(k) < 30 + M\delta_1(k), \]

• Logic constraints become mixed integer linear constraints
Mixed Integer Quadratic Programming (MIQP)

\[
\min_{\mathbf{u}, \delta} \sum_{k=0}^{K} \left\| \mathbf{x}(k) - \mathbf{x}_r(k) \right\|_Q^2 \\
+ \left\| \delta(k) - \delta_r(k) \right\|_S^2 + \left\| \mathbf{u}(k) \right\|_R^2,
\]

subject to

known \( \mathbf{x}(0) \),
vehicle dynamics,
dynamic constraints,
\[
C \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_r \\ \delta \end{bmatrix} \leq D.
\]

Note that the cost function is quadratic and all constraints are linear, thus the problem is MIQP.

MIQP is NP while efficient branch-and-bound algo exist: CPLEX, GUROBI
Example applications

Speed bump
Obstacle avoidance
Overtaking
Example applications

Lane change
A new viewpoint for optimization-based trajectory planning using MIQP

• We consider logical constraints induced by traffic rules and multiple maneuver variants

• We formulate logical constraints as propositional logics and transform them into mixed integer inequalities

• We formulate a MIQP problem to find the global optimal trajectories under these constraints

• It can serve as a basis for cooperation among vehicles